Division **E**

Mathematical Olympiads

FEBRUARY 3, 2009

MATH OLYMPIAD

for Elementary and Middle Schools

Contest **4**

4A *Time: 4 minutes*

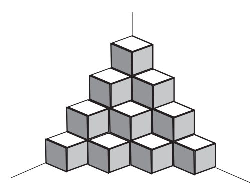
A digital clock shows 2:35. This is the first time after midnight when all three digits are different prime numbers. What is the last time before noon when all three digits on the clock are different prime numbers?

4B *Time: 5 minutes*

The only way that 10 can be written as the sum of 4 different counting numbers is 1 + 2 + 3 + 4. In how many different ways can 15 be written as the sum of 4 different counting numbers?

4C *Time: 5 minutes*

The tower shown is made of congruent cubes stacked on top of each other. Some of the cubes are not visible. How many cubes in all are used to form the tower?



4D *Time: 6 minutes*

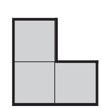
Hannah gives clues about her six-digit secret number:

Clue 1: It is the same number if you read it from right to left.
Clue 2: The number is a multiple of 9.
Clue 3: Cross off the first and last digits. The only prime factor of the remaining four-digit number is 11.

What is Hannah's six-digit number?

4E *Time: 7 minutes*

The L-shape pictured is formed from three squares, each 1 cm on a side. Five of these L-shapes are placed next to each other to form a figure. What is the least possible perimeter of the figure they form, in cm?



Please fold over on line. Write answers on back.

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SOLUTIONS AND ANSWERS

4A *Items in parentheses are not required.*

4A *Strategy: List the single-digit primes.*
The single-digit prime numbers are 2, 3, 5, and 7. Select the 3 greatest numbers from this list and write them from largest to smallest. **The last time before noon when all 3 digits are prime is 7:53.**

FOLLOW-UP: *At how many times between midnight and noon will the digits be 3 different primes?* [18]

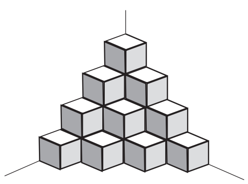
7:53 (AM)

4B *Strategy: Make an organized list.*
List the ways four different numbers can add to 15. Starting with the largest number reduces the number of trials necessary. Because 3 + 2 + 1 = 6 is the least possible sum for 3 of the numbers, the greatest can't exceed 15 – 6 = 9.
15 = 9 + 6 = 9 + 3 + 2 + 1
15 = 8 + 7 = 8 + 4 + 2 + 1
15 = 7 + 8 = 7 + 5 + 2 + 1 or 7 + 4 + 3 + 1
15 = 6 + 9 = 6 + 5 + 3 + 1 or 6 + 4 + 3 + 2
15 can be written as the sum of four different counting numbers in 6 different ways.

6 (ways)

4C **METHOD 1:** *Strategy: Count horizontally, from the top layer down.*
Make a table that counts cubes separately for each layer. In each case, add the number of hidden cubes to the number of visible cubes.

Layer	Cubes in layer (visible + hidden)
1 (top)	1 = 1
2	2 + 1 = 3
3	3 + 3 = 6
4 (bottom)	4 + 6 = 10



1 + 3 + 6 + 10 = **20 cubes are used to form the tower.**

METHOD 2: *Strategy: Count vertically, stack by stack.*
This table counts cubes separately for each stack (column), from the shortest to the tallest. Both hidden and visible cubes are counted.

Height of stack	No. of stacks	Total cubes by height
1	4	4
2	3	6
3	2	6
4	1	4

20 cubes are used to form the tower.

20 (cubes)

4D **513315** (or 513,315)

4E **16** (cm)

FOLLOW-UPS on other side.

Follow-Ups: (1) How many cubes in all would be in a similar tower of 5 layers? 6? 7? [35, 56, 84] *(2) Suppose the bottom layer of a similar tower has 91 cubes. How many layers would there be?* [13]

4D *Strategy: Consider the clues one at a time, starting with the most restrictive.*

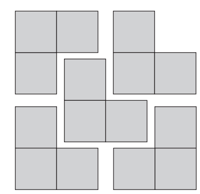
Clue 3: The only prime factor of the 4-digit number is 11, so the number = 11×11 or $11\times11\times11$ or $11\times11\times11\times11$, etc. Of these, only $11\times11\times11= 1331$ has 4 digits, so the middle 4 digits are 1331.

Clue 1: The number reads the same right to left, so the first and last digits are the same. Call the number A1331A.

Clue 2: The number is a multiple of 9, so the sum of its digits is a multiple of 9. $A + 1 + 3 + 3 + 1 + A = A + 8 + A$ must equal 9 or 18. No digit A satisfies $A + 8 + A = 9$, but if $A + 8 + A = 18$, $A = 5$. **Hannah's number is 513315.**

4E *Strategy: Make the figure as compact as possible.*

The area of the given L-shape is 3 sq cm. A figure made up of 5 L-shapes has an area of 15 sq cm. The fact that all the angles in each shape are right angles suggests trying to pack the 5 shapes into a square of area 16 sq cm. This can be done as shown in the figure. **The least possible perimeter is** $4 + 4 + 3 + 3 + 1 + 1 =$ **16 cm.**



NOTE: *Other Follow-Up problems related to some of the above can be found in our two contest problem books and in "Creative Problem Solving in School Mathematics." Visit* www.moems.org *for details and to order.*